

Visibility bound caused by a distinguishable noise particle

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We investigate how distinguishability of a “noise” particle degrades interference of the “signal” particle. The signal, represented by an equatorial state of a photonic qubit, is mixed with noise, represented by another photonic qubit, via linear coupling on the beam splitter. We report on the degradation of the “signal” photon interference depending on the degree of indistinguishability between “signal” and “noise” photon. When the photons are principally completely distinguishable but technically indistinguishable the visibility drops to the value $1/\sqrt{2}$. As the photons become more indistinguishable the maximal visibility increases and reaches the unit value for completely indistinguishable photons. We have examined this effect experimentally using setup with fiber optics two-photon Mach-Zehnder interferometer.

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I. INTRODUCTION

The key property of the quantum world is the existence of superpositions of states. This property plays a crucial role in quantum information transfer and processing. However, many systems cannot preserve coherent superpositions for long time due to decoherence [1, 2]. This process of decoherence may have several reasons: It can arise from fluctuations of external macroscopic physical parameters of the system (dephasing) [3] or it can appear as the result of a coherent coupling between the system and the environment [4, 5] (experimentally tested in Ref. [6]) or it can be evoked by mixing the quantum system with some other system representing noise. In this paper we will focus on the latter. Contrary to the standard model of decoherence our mechanism is not based on coherent interaction between the system and environment [2, 4]. No spontaneous emission from the system [7] or collision to the environment appear [8]. The only relevant source of decoherence is the added noise and its distinguishability from the signal [9]. The problem of noise is of a special interest for quantum communication [10]. The physical scenario we describe is relevant for boson like particles, to extend it for fermions their specific properties had to be taken into account [11].

II. THEORETICAL FRAMEWORK

The simplest system in quantum information is a qubit. In many applications and experiments (like in quantum key distribution, tests of Bell inequalities, etc.) only a subset of all possible states of a qubit is used. Namely, the set of equatorial states (they lie on the equator of the Bloch sphere) represented by a coherent superposition $[|0\rangle + \exp(i\varphi)|1\rangle]/\sqrt{2}$ of two basis states $|0\rangle$ and $|1\rangle$. Phase φ is used to encode information. We investigate how noise can affect the coherence of equato-

rial state of qubit. We suppose no fluctuation of phase and no interaction with the environment (therefore no entanglement can appear between the qubit and the environment). Let us represent our qubit by a single particle distributed between two modes A and B . Its equatorial state can be described as $|\Psi\rangle_{AB} = [|1, 0\rangle_{AB} + \exp(i\varphi)|0, 1\rangle_{AB}]/\sqrt{2}$, where 0 and 1 represent the number of particles. The noise is caused by another particle in mode B' which can be confused with the original particle. We suppose that modes B and B' are *principally* distinguishable but *technically* indistinguishable. It means our detectors cannot discriminate them. We will consider the situation when a “noise” particle is created in mode B' and subsequently one particle is annihilated either from mode B or B' (our device cannot distinguish between them).

To examine this situation experimentally, we use a Mach-Zehnder (MZ) interferometer and a source of photon pairs (Fig. 1). The “noise” photon with variable distinguishability from the “signal” one is fed into one arm of the interferometer. If the “noise” particle was created in mode B it would be *principally indistinguishable* from the “signal” particle in mode B . For a bosonic field the total quantum state after the creation process $a_B^\dagger|\Psi\rangle_{AB}$ reads $|\Psi'\rangle_{AB} = [|1, 1\rangle_{AB} + \sqrt{2}\exp(i\varphi)|0, 2\rangle_{AB}]/\sqrt{3}$. It has full coherence: the phase information is fully preserved. The indistinguishable photon can be eliminated by the act of annihilation $a_B|\Psi'\rangle_{AB}$ ending up with state $|\Psi''\rangle_{AB} = [|1, 0\rangle_{AB} + 2\exp(i\varphi)|0, 1\rangle_{AB}]/\sqrt{5}$. It can be further probabilistically converted to the original state of the signal qubit applying attenuation $\eta_B = 1/4$ in mode B . Since the both particles are indistinguishable, it does not matter which one has been actually taken out. The visibility of interference can reach unity again. We define the single photon visibility by the standard formula $V = (P_{\max} - P_{\min})/(P_{\max} + P_{\min})$ where $P_{\max} = \max_{\varphi} P(\varphi)$, $P_{\min} = \min_{\varphi} P(\varphi)$ with $P(\varphi)$ being the probability to detect photon at detector D depending

on phase φ .

If the “noise” particle is *principally distinguishable* from the “signal” one, it can be described by creation operation $a_{B'}^\dagger |\Psi\rangle_{AB} |0\rangle_{B'}$. In principle, it could be filtered out, because it differs in its properties from the “signal” particle. But, quite typically, our filters are not selective enough to enable it. If the disturbing particle is only *technically indistinguishable*, the total state of the system is transformed to $|\Phi'\rangle = [|1, 0, 1\rangle_{ABB'} + \exp(i\varphi) |0, 1, 1\rangle_{ABB'}] / \sqrt{2}$ after the creation in mode B' . Because we are not able to discriminate modes B and B' , to remove a single particle we just randomly annihilate a single particle either from B or B' (with no prior knowledge this strategy is fully symmetrical). Further, without any access to mode A , this process can be described by two “subtraction” operators: $S_1 = a_B \otimes 1_{B'}$ and $S_2 = 1_B \otimes a_{B'}$, acting with equal probabilities. Applying these operators on $\rho' = |\Phi'\rangle\langle\Phi'|$, i.e. $S_1 \rho' S_1^\dagger + S_2 \rho' S_2^\dagger$, one gets the resulting mixed state $\rho'' = 2/3 |\Psi\rangle_{AB} \langle\Psi| \otimes |0\rangle_{B'} \langle 0| + 1/3 |00\rangle_{AB} \langle 00| \otimes |1\rangle_{B'} \langle 1|$. Because our detectors cannot distinguish whether the particle came from mode B or B' , the visibility of interference is now $V'' = 2/3$. It can be probabilistically enhanced by a proper attenuation $\eta_B = \eta_{B'}$ in modes B and B' . This transforms the total state to

$$\rho''' = \frac{1}{2\eta_B} [|100\rangle\langle 100| + \eta_B |010\rangle\langle 010| + \eta_B |001\rangle\langle 001| + \sqrt{\eta_B} (e^{i\varphi} |010\rangle\langle 100| + e^{-i\varphi} |100\rangle\langle 010|)] \quad (1)$$

Clearly, if $\eta_B = 1/2$ one balances the probability of having a particle in mode A with the probability of having it either in mode B or B' . Then the visibility is maximal and reaches the value

$$V_{dis} = \frac{1}{\sqrt{2}}. \quad (2)$$

In comparison to the previous case of indistinguishable particles, this reduction of visibility represents a fundamental impact of the principal distinguishability of the “signal” and “noise” particles. The loss of coherence is a result of an elementary ignorance of our measurement apparatus. There is a difference between our result and the one reported in [12], where visibility completely vanishes for distinguishable photons. According to Englert’s inequality $V^2 + K^2 \leq 1$ [13, 14], this elementary visibility reduction corresponds to overall which-way knowledge $K < 1/\sqrt{2}$ accessible in the experiment.

If N completely distinguishable particles are created simultaneously in modes associate to mode B and, subsequently, N particles are simultaneously annihilated in these modes and mode B , visibility rapidly decreases with increasing N : $V_{dis}(N) = \frac{1}{\sqrt{N+1}}$. On the other hand if the particles are created and annihilated subsequently, i.e. after any single-particle creation a single particle is always annihilated, visibility is decreasing even faster: $V_{dis}(N) = \left(\frac{1}{\sqrt{2}}\right)^N$.

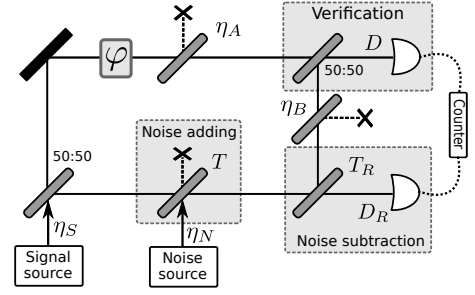


FIG. 1: The simple scheme of Mach-Zehnder two-photon interferometer illustrates the process of interference measurement.

III. EXPERIMENTAL IMPLEMENTATION

In the experiment, mode A is represented by the upper arm of the interferometer (Fig. 1) and modes B, B' by the lower arm (they are distinguishable in time domain). Creation and annihilation process is emulated by a beam splitter. The action of the beam splitter can be described by a unitary operator $U = \exp[\theta(a^\dagger a_{aux} - a_{aux}^\dagger a)]$, where “aux” denotes the auxiliary mode and θ is related to the intensity transmittance by the formula $T = \cos^2 \theta$. If $|\theta| \ll 1$ and there is a proper state in the auxiliary mode, U well approximates action of creation, a^\dagger , or annihilation, a , operators. The coincidence measurement guarantees that only those situations are taken into account, when exactly one photon is annihilated and one photon is detected at the output of the interferometer.

To be realistic and comparable with experimental results, the theoretical prediction must take into account a finite coupling (i.e. transmittances and reflectances of beam splitters) as well as all insertion losses. In Fig. 1 the signal source generates single photons which are coupled, with efficiency η_S , to the interferometer and afterwards they are split equally likely to the upper or lower arm of the interferometer. In the upper arm we can set the phase shift φ and adjust losses by a beam splitter with transmissivity η_A (to achieve maximum interference). The noise source feeds single photons into the lower arm of the interferometer with coupling efficiency η_N and then the photons are coupled by a beam splitter with transmissivity T to the signal photons. The internal losses of the interferometer are modelled by a beam splitter with transmissivity η_B . As was indicated in the introduction, in order to subtract noise we suggested to annihilate one photon from the lower arm of the interferometer. This is accomplished by inserting another beam splitter which transmits photons with ratio T_R to another detector D_R . To evaluate the effect of the noise subtraction we measure visibility of the signal from detector D conditioned on the detection event from the detector D_R . Calculation for fully indistinguishable “noise” photon leads to

visibility

$$V_{ind} = \frac{4\sqrt{\eta_A\eta_B T(1-T_R)}}{\eta_A + 4\eta_B T(1-T_R)}. \quad (3)$$

Optimizing the values of free parameters we can reach

$$V_{ind}^{max} = 1 \quad (4)$$

for $\eta_A = \eta_B T$, $T_R = 3/4$. The perfect visibility is achieved, as was predicted also in the previous discussion of simplified model. In the fully distinguishable scenario the visibility reads

$$V_{dis} = \frac{2\sqrt{\eta_A\eta_B T(1-T_R)}}{\eta_A + 2\eta_B T(1-T_R)}. \quad (5)$$

If we optimize the values of free parameters we can reach

$$V_{dis}^{max} = \frac{1}{\sqrt{2}} \quad (6)$$

for $\eta_A = \eta_B T$, $T_R = 1/2$. We can see the drop in visibility to the value $1/\sqrt{2}$.

In practice, the photons are partially indistinguishable. We can describe this situation as a mixture of the two limit cases: With probability p the “signal” and “noise” photons are principally indistinguishable, otherwise they are principally distinguishable! We have repeated the calculation of visibility (similarly to the previous extreme cases) for the above defined mixture of an indistinguishable and distinguishable “noise” photon. The visibility reads

$$V(p) = \frac{2(1+p)\sqrt{\eta_A\eta_B T(1-T_R)}}{\eta_A + 2(1+p)\eta_B T(1-T_R)}. \quad (7)$$

Optimizing the values of free parameters the visibility reaches its maximum

$$V^{max}(p) = \sqrt{\frac{1+p}{2}}, \quad (8)$$

for $\eta_A = \eta_B T$, $T_R = (1+2p)/[2(1+p)]$. The more distinguishable is the noise photon from the signal photon the lower visibility we can obtain. The transmissivity T , determining the strength of the coupling between the noise and signal photon, has no influence on the visibility. We have used a setup depicted in Fig. 2 to experimentally test the theoretically predicted visibility (8) for the two extreme cases of distinguishability. The key part of the setup is the MZ interferometer build of fiber optics that allows us to simply control transmissivities T and T_R via variable-ratio couplers (VRCs) within the range 0-100%. Signal and noise photons are created by type-I degenerate spontaneous parametric down-conversion in a nonlinear crystal of LiIO_3 pumped by a cw Kr-ion laser (413 nm). Photons from each pair are tightly time correlated, have the same polarization and the same spectrum centered at 826 nm. The degree of distinguishability of

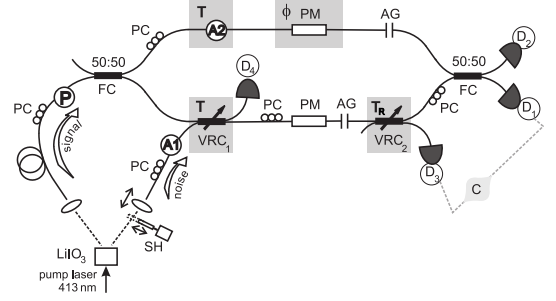


FIG. 2: Experimental setup. Shutter (SH), polarization controller (PC), polarizer (P), attenuator (A), phase modulator (PM), adjustable air-gap (AG), fiber coupler (FC), variable-ratio coupler (VRC), detector (D).

signal and noise photons can be tuned changing the time delay between their wave-packets at VRC_1 . This is realized moving a motorized translation stage connected to the fiber coupling system at the “noise” side. All other characteristics of the photons are identical.

Before the measurement the source of photon pairs is adjusted by optimizing the visibility of two-photon interference at VRC_1 with splitting ratio 50:50. The visibility of Hong-Ou-Mandel (HOM) dip [15] reaches typically values about 98%. Then the equality of intensities of signal and noise coupled to the fibers is verified measuring the count rates at detectors D_3 and D_4 . The count rates of the noise photons at these detectors have to be double in comparison with the count rates of the signal photons. The required signal to noise ratio is then set tuning the intensity transmissivity T of VRC_1 . According to the theoretical proposal, the transmissivity of the upper arm of the MZ interferometer is also set to the value T using attenuator A_2 . At this point we unbalance the interferometer setting the optimal transmissivity T_R of VRC_2 . This variable ratio coupler separates a part of the light from the lower interferometer arm for a post-selection measurement on the detector D_3 . It should be stressed that these additional losses are not compensated in the upper arm of the MZ interferometer.

Technical remarks:

- (i) To accomplish proposed experiment, only one phase modulator in the upper arm of the MZ interferometer is needed. The second phase modulator in the lower arm just guarantees the same dispersion in both interferometer arms. This trick allowed to increase the visibility approximately by 13 % to 94%.
- (ii) All used detectors are Perkin-Elmer single-photon counting modules. To implement the post-selection measurement the signals from detectors are processed by coincidence electronics with a coincidence window of 2 ns.
- (iii) The absolute phase in optical fibers is influenced by temperature changes. Resulting undesirable phase drift is reduced by a thermal isolation of MZ interferometer and the residual phase drift is compensated by an active

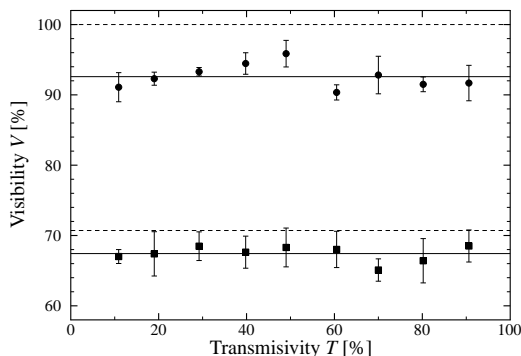


FIG. 3: Visibility V as a function of the transmissivity T . Symbols denote experimental results; squares correspond to the case of distinguishable photons and circles to the case of indistinguishable photons. Solid lines are fits of measured data and dashed lines are theoretical predictions.

stabilization.

IV. RESULTS

The aim of the experiment is to show how the visibility of the signal photon is affected by a distinguishable and indistinguishable “noise” photon after the “noise subtraction”. We measured coincidence rate C between detectors D_1 and D_3 . Intensity transmissivity of VRC_2 was adjusted so that the visibility of coincidence rate C was maximal, i.e., $T_R = 1/2$ for distinguishable photons and $T_R = 3/4$ for indistinguishable photons. The visibility of C was measured for different values of the transmissivity T . $T = 100\%$ represents no added noise case, $T = 0\%$ means that the signal photon can not pass through the lower arm of the MZ interferometer. These two limit cases could not be measured, because the coincidence rate C vanishes.

Figure 3 shows visibilities of the coincidence-rate interference patterns. Each interference-fringe measurement, consisting of 41 phase-steps, was repeated five times. Coincidence-rate measurement for each phase step takes typically 3 s. After each three-second measurement period the phase was actively stabilized. The results displayed in Fig. 3 support the theoretical prediction that visibility V does not depend on T . Obtained mean value of visibility is $67.4 \pm 1.1\%$ for distinguishable noise photons (the theoretical value is $1/\sqrt{2} \approx 70.7\%$) and $92.6 \pm 1.7\%$ for indistinguishable noise photons (the theoretical value is 100%). Shown error bars represent statistical errors. Systematic shifts of the values are due

to experimental imperfections. It should be noted that the measurement with distinguishable noise photons is more robust. In the case of indistinguishable photons the visibility is very sensitive to fluctuations of the time overlap of the two photons. Due to this fact, the visibility measured with distinguishable photons lies closer to the theoretical limit.

V. CONCLUSIONS

We have observed how the noise represented by an additional distinguishable particle can degrade interference. It is known that as a consequence of decoherence events very fast sudden death of entanglement can happen and Bell-inequality violation can disappear [18]. So, let us imagine now that instead of a single signal photon entering the interferometer through the input beam splitter we have a photon inside the interferometer which is a member of a pair maximally entangled in spatial modes. So we have two maximally entangled qubits, one of them goes through our noisy “channel” followed by the “noise subtraction” and finally it is measured in the basis consisting of two orthogonal equatorial states. Pairs of maximally entangled qubits can be used to test the exclusivity of quantum mechanics. If they are measured locally in proper bases (which can be fully constructed from the equatorial states) the results violate Bell (or CHSH) inequalities [17]. However, once one of the qubits is sent through our channel and once the detectors are not able to distinguish between modes B and B' no violation of the Bell inequality is observed. To reveal the violation, a measurement outside the equatorial plane has to be performed. It is not surprising, since the considered decoherence process is basis dependent. It fully disturbs only the results of measurements in the equatorial plane. Tittel et al. [16] used energy-time entangled photons to test Bell-inequality violation under the dephasing process in an optical fiber and they have proved that the necessary condition to observe the violation reads $V > 1/\sqrt{2}$. In the decoherence process described in this paper the maximal visibility (in the case of “distinguishable” noise) reaches just this boundary value.

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